Digital Surface Modeling and Volumetric Analysis Techniques Applied to the Measurement of Plan-View Earthwork Quantities

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October, 1993

Author's Note

I completed the literature research and writing for this paper nearly 30 years ago but, with numerous household moves and computer upgrades in the years since, I managed to lose the digital files for the original paper. Miraculously, I did retain a hard copy of the paper (a poor-quality scanned raster copy of which has been linked for a dozen years on my website at **www.earthworksoftwareservices.com/downloads/whitepaper.pdf**). After long intending to reconstruct this paper's digital files in order to produce a clean vector PDF copy for the website, I'm happy to report that I finally devoted a block of time for completion of the task.

This reconstructed copy of my original paper has been lightly edited—mostly to fix typographical errors, including a few incorrectly numbered footnotes, but two supporting figures from my original research sources have been added that were previously excluded from the 1993 paper. Anyway, this reconstructed paper faithfully honors the text and intent of the original document (including the now outdated hardware references, which today's reader can ignore with a chuckle).

Although it's dated, this 30-year-old paper may still offer information of interest to readers involved in modeling and quantifying site earthwork: **Pages 2-6** provide a review of the old ***Manual Methods for Measuring Earthwork*** (grids, cross sections and contour slices), including common adjustments made for the resulting earthwork quantities; **Pages 7-18** discuss the theory and methods used to create ***Digital Surface Models***, including data entry, surface modeling methods (mesh grids, triangulated irregular networks and composite variations thereof) and the importance of incorporating breaklines for accurate surface models; finally, **Pages 19-23** discuss the primary methods used for calculating digital ***Surface-to-Surface Volumes*** (grid cells, cross sections and triangular isopaches).

Thank you for reading!

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Introduction

Earthmoving requirements for construction projects, such as those encountered in commercial, industrial, or residential developments, can be extensive and complex. When these projects are under private contract (and some government contracts) the earthwork is normally bid on a *lump sum* or *firm quote* basis, under which the earthmoving contractor assumes a high degree of risk.[1] Given both narrow profit margins and high earthmoving operating costs, the contractor can best reduce this risk by performing an accurate pre-bid earthwork estimate. Traditionally, earthwork estimates have been calculated by hand, at the cost of tens (sometimes hundreds) of expert man hours. For the typical site work contractor (whose work can include earthmoving, grading, asphalt and concrete paving, and underground utilities), estimating earthwork quantities is the most time consuming takeoff activity.[2]

Earthwork takeoff is extremely time consuming because it is so complex: a proposed design topography, consisting of irregularly shaped areas and variable slopes, elevations, and offsets is being compared to an existing topography of convexities, concavities, ridges, swales, and planes. Variable sub-surface conditions add yet another dimension of complexity. It follows, then, that the required manual calculations are tedious (at best) or recondite (often). Fortunately, the last ten years [1983-1993] have witnessed the rapid development of microcomputers and digitizers, as well as the concomitant introduction of various software packages designed to aid in the calculation of complex earthwork quantities. Unfortunately, these systems have suffered from inaccuracies, poor performance, or too high a price.

After reviewing the various manual techniques used in calculating earthwork volumes, this paper will discuss current digital methods and, finally, conclude by outlining the author's specification for a digital earthwork modeling and analysis system which is both accurate and affordable.

[1] Atherton and Alves, 14; Atcheson, 5; Nichols, 11-48 - 11-49.
[2] Atherton and Alves, 82.

Manual Methods for Measuring Earthwork

Earthwork for the nonlinear types of projects described above is typically represented in two dimensions on a scaled site grading map. The site grading map is presented as a *plan view*[3] and must sufficiently represent both existing and design elevations so as to allow completion of the specified work.[4] Existing and design elevations are represented in plan view by lines of equal elevation (e.g., contour lines and building outlines), lines of changing elevation (e.g., ridges, swales, curb lines, tops and bottoms of slopes), and spot elevations.[5] It is from this information that earthwork quantities must be measured before estimating the cost of earthwork for any particular project. There are three generally accepted methods for manually measuring earthwork volumes[6]: (1) grid cells, (2) cross sections, and (3) contour slices. Each of these methods will be discussed here.

Grid Cells

In its simplest form, this method superimposes a grid of equally spaced, perpendicular lines on the grading plan. Existing and design elevations are then interpolated at each grid intersection by noting its spatial relationship to elevation data delineated on the plan. Once existing and design elevations have been determined for all grid intersections, the depth of cut (or fill) can be obtained by calculating the difference between the existing and proposed elevations at each intersection. Within each grid cell, an average depth of cut (or fill) is calculated from the depths determined at that cell's four grid intersections. The average depth of cut (or fill) is then multiplied by the grid cell's horizontal surface area to compute that cell's volume of cut (or fill). Total site volumes of cut and fill are obtained by summing the individual cut and fill values for all grid cells.[7]

The grid method can be fast and reasonably accurate for sites with relatively flat existing and design topographies. This method can be used on sites when existing and/or design topography is not flat, but the grid line spacing must be reduced so as to more accurately sample the topography, while the number of calculations required increases geometrically. Therefore, the grid method is inappropriate for many sites.

[3] A planimetric view, as opposed to the *plan and profile views* normally used in representing roadwork and other linear-type projects.
[4] Zimmerman, 278.
[5] Atcheson, 9; Capachi, 17-36; Kennie, 162; Zimmerman, 272-278.
[6] Atherton and Alves. 82; Cardon, 678.
[7] Atcheson, 13-21; Atherton and Alves, 82; Cardon, 685-689; Schultz, 198.

Cross Sections

With the cross-section method, a base line -- usually centered longitudinally through the site -- is superimposed on the site grading plan. Beginning at one end of the base line, and perpendicular to it, cross-section lines are then superimposed on the plan at specified intervals. The length of the interval between adjacent cross-section lines is dictated by changes in the existing and design topographies as represented by the grading plan: areas of changing topography require more cross-section lines, while constant topography requires fewer cross-section lines. Ideally, a cross-section line is plotted at every major existing and design elevation grade change.[8] Once the base line and required cross-section lines have been identified and plotted on the plan, scaled cut and fill cross sections corresponding to each cross-section line can be drawn by plotting the respective existing and design elevation profiles on graph paper. For any given cross-section line, the earthwork estimator notes the horizontal position of existing elevation contours, spot elevations, and grade breaks intersected by the cross-section line, plotting this data on the graph paper as a series of points, and producing an existing elevation profile by connecting the points with a plotted line. A design elevation profile is produced using the same technique and the cut and fill cross section at that station is complete. The end areas (vertical surface areas) of cut and fill at each completed cross section are then measured with a planimeter.[9]

Once all cross sections for the site have been plotted to graph paper, and their end areas measured, cut and fill volumes are calculated using either the average-end-area or prismoidal formula. With the average-end-area formula, cut and fill volumes between adjacent cross sections are calculated by averaging the respective cut and fill end areas and multiplying the result by the perpendicular distance between the adjacent cross sections. By repeating the process for all sets of adjacent cross sections and totaling the results, total site cut and fill volumes are obtained.[10]

Normally, volumes calculated using the average-end-area method offer acceptable accuracy and are legally recognized.[11] There is, however, a caveat: the average-end-area method is accurate only when the corresponding end areas of adjacent cross sections have equal surface areas. As the difference in surface area of the respective end areas increases, the average-end-area formula will yield volumes increasingly larger than the corresponding true volumes. Where end areas differ greatly, this problem can be minimized either by increasing the number of cross sections taken in a given interval (and

[8] Atcheson, 61.
[9] Atherton and Alves, 83-84; Cardon, 679; Nichols, 2-21; Zimmerman, 275-276.
[10] Atcheson, 21-23; Cardon, 679-680.
[11] Cardon, 680.

thereby reducing the end-area differences between adjacent cross sections in that interval) or by use of the prismoidal formula.[12]

The prismoidal formula, based on Simpson's Rule for approximating the area below a curve,[13] provides accurate measures even when the end areas of adjacent cross sections differ widely. But it is time consuming and difficult to use. It is normally reserved for special, high-cost applications such as measuring quantities for rock excavation and concrete structures.[14] Regardless of the formula used to calculate cut and fill volumes, the cross-section method offers some important advantages. It is particularly effective due to the visual feedback it provides.[15] The relationship between the existing and design topographies is immediately apparent and any errors are rendered obvious. Cross sections are very effective when preparatory or intermittent work is required: profiles representing topsoil stripping, over excavation and recompaction, and pavement sections can be plotted at each cross section as needed (quantities for each of which can then be measured via planimetering).[16] Cross sections are also useful for the periodic documentation of borrow pit quantities,[17] and their accuracy and legal acceptability has already been noted.

Contour Slices

The contour-slice method is conceptually similar to the cross-section method. The cross-section method measures areas of vertical planes applied to horizontal distances to calculate volumes, whereas the contour-slice method measures areas of horizontal planes applied to vertical distances. The horizontal surface areas bounded by selected existing and design contour lines are measured with a planimeter and volumes are calculated using the average-end-area method. Contour slices are commonly used to calculate volumes when deep excavations are encountered (e.g., open-pit mining) or when rolling terrain and mounds (e.g., parks and golf courses) render grids and cross sections impractical by requiring very close grid or cross-section line intervals.[18]

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[12] Cardon, 679-681.
[13] Larson and Hostetler, 494-499.
[14] Atcheson, 24; Cardon.681-685.
[15] Atherton and Alves, 83.
[16] Atherton and Alves, 84.
[17] Fish, 765-766.
[18] Atcheson, 62-67; Atherton and Alves, 84; Cardon, 689-690.

Adjustments

Regardless of which method is used to determine cut and fill volumes, certain adjustments must be made to the volumes in order to properly analyze earthwork requirements for any given site. These adjustments normally include (1) topsoil stripping quantities, (2) strata subtotals, (3) shrink/swell factors, (4) subsidence factors, and (5) horizontal/vertical subtotals.

Because vegetation and organically contaminated soils are incompatible with engineered fills, topsoil stripping is a usual requirement prior to cut and fill operations. Stripping depths can range from several inches to several feet and are reflected in the cut and fill volumes by increased quantities of fill and decreased quantities of cut. Topsoil stripping complicates manual earthwork calculations by shifting the zero or *daylight* line,[19] thereby shifting the horizontal limits of cut and fill. Care must also be taken when manually calculating stripping quantities from horizontal measures on steep slopes, as the true quantities can be significantly larger.[20]

Strata subtotals are needed on sites where dissimilar natural subsurface materials require special handling and/or pricing. The depth of different strata layers are normally given in tabular form in a geotechnical report and cross referenced to test borehole locations on the site plan. The depth of strata materials for any location on the site can be interpolated from the test boreholes either by *eyeballing* (mental interpolation) or through more sophisticated means, such as estimating strata planes using coordinate geometry and matrix algebra. In either case, the ultimate goal is to subtotal cut volumes by subsurface material type.[21]

Shrink/swell factors are needed in order to make adjustments for conversions between the three fundamental soil-volume conditions: (1) bank (natural/undisturbed), (2) compacted (engineered), and (3) loose (hauled/stockpiled). Normal earthwork operations see soil move through the three fundamental states as it is cut (bank state), transported and/or stockpiled (loose state), and finally filled (compacted state). The density of any given soil type is different in each of these three states. Typically, but not always, density is highest in a compacted state and lowest in a loose state.[22] The general method of determining the shrink/swell factor is to divide the specified compacted fill density by the bank density of

[19] The zero cut/fill line separating contiguous cut and fill areas.
[20] Atcheson, 49-60, 179-184; Atherton and Alves, 37, 82; Nichols, 10-15 - 10-16.
[21] Atcheson, 163-177; Nichols, 11-49.
[22] Rock, for example, usually has a higher density in a bank state than in a compacted state. As rock is broken into pieces during excavation its volume expands due to voids around the broken pieces; these voids are not completely eliminated when the broken rock is placed as compacted fill, thus lowering its density relative to the bank state.

the corresponding cut material.[23] If the answer is greater than one, the material will shrink as fill; if the answer is less than one, the material will swell. In either case, the resulting factor is multiplied by the raw volume of fill. This adjustment converts the fill from a compacted volume to a bank volume, allowing direct comparison with the bank volume of available cut. Without such a conversion, an analysis of the balance between cut and fill can yield an erroneous import/export volume estimate.[24]

Subsidence factors (also known as settlement or consolidation factors) are a special application of shrink factors. Shrinkage can be experienced in natural soils below compacted fills. This can occur during construction due to the weight of earthmoving equipment, causing settlement of several tenths of a foot (and increasing the required amount of fill), or it can occur after construction due to the weight of fill soils and structures thereon. The latter case is sometimes compensated for by placing a surcharge load on the fill areas for several weeks or months prior to construction. It is also common practice for engineers to require scarification and pre-compaction of fill areas prior to placing any fill. In all these cases, some adjustment should be made to the fill volume to compensate for subsidence. The adjustment can be made by increasing the volume of fill by the depth of settlement. If more exact information is lacking, settlement depth can be calculated by multiplying the compaction depth by the shrink factor and dividing the result by two, which simply averages maximum compaction at the top and zero compaction at the bottom.[25]

The requirements for horizontal and vertical subtotals are a function of how a project must be bid and constructed. Horizontal subtotals are very common and are needed when projects require pricing by area or phase. Subtotals by area can also be helpful for checking production and measuring haul distances. Vertical subtotals are useful for deep cuts and fills, where production costs may vary by depth.[26]

[23] These densities are normally found in a geotechnical report and expressed as a percent of theoretical maximum density based on a Standard or Modified Proctor Test.
[24] Atcheson, 73-77; Nichols, 2-49 - 2-50, 8-41.
[25] Atherton and Alves, 38; Nichols, 2-50, 8-41.
[26] Atherton and Alves, 85.

Digital Surface Models

Digital surface modeling (DSM) techniques have evolved continuously over the last twenty years [1973-1993]. In its early years, DSM required large and powerful computers. But recent, rapid advances in computer miniaturization and processing power have been paralleled by equally rapid improvements in DSM techniques and software.[27] After a brief description of digitizing methods, this section will discuss how current methods allow the generation of DSMs for existing and design topographies using digitized data.

Digitizing Plan-View Data

Before DSM techniques can be applied to the analysis of plan-view earthwork, elevation data must be converted from a two-dimensional analog format (paper plans) to a three-dimensional digital format.[28] There are two alternatives for digitizing elevation data from grading plans: manual digitizing and raster scanning.[29]

Manual digitizing can be accomplished using one of many widely available, and relatively low cost, electronic digitizing tablets. The paper plan is placed on the digitizing tablet, which contains a grid of x-y positioning wires. A combined cursor (field coil) and numeric keypad is used to manually enter elevations (z values) and trace the corresponding elevation lines from the plan. X-y position data is registered automatically from the cursor's field coil position over the tablet's wire grid.[30] Position and elevation data from the digitizing tablet is recorded by a computer and appropriate software.[31] An initial two-point planimetric scaling procedure allows the software to automatically convert data-point coordinate values from the digitizer's coordinate system to that of the out-put system (the paper plan's reference system).[32] The end result is a vector data file of properly scaled x-y-z triplets ready for conversion to a DSM.

Raster scanning circumvents manual digitizing by using either a drum or flatbed scanner to automatically read contour and other line data from paper plans.[33] While raster scanning may appear to offer a faster alternative to manual digitizing, it is not perfect.

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[27] Milne [*Computer Graphics*], vii; Petrie and Kennie, xiii; Webb, 73.
[28] Although much civil design work is now completed using a computer aided design (CAD) system, manual digitizing and/or raster scanning is obviated by the fact that many design firms are unwilling to release a project's CAD files to the contractor for pre-bid analysis.
[29] Petrie [*Terrain*], 87.
[30] Petrie [*Terrain*], 91-93.
[31] For a BASIC program source code listing of a digitizer input subroutine, see Milne [*Computer Graphics*], 27-28.
[32] Douglas [*XYNIMAP*], 4.
[33] Petrie [*Terrain*], 97-99.

Raster scanning produces a non-vector data structure which, typically, must first be converted to a vector format before final conversion to a DSM.[34] This raster-to-vector data conversion requires algorithms for both line extraction and line thinning, and consumes "vast" amounts of data processing time. And, because raster scanning does not record z values, raster-derived data must be manually edited for elevation tagging. Finally, raster scanning is capital intensive and perhaps better suited to a commercial digitizing bureau than to the in-house needs of the individual civil engineer, landscape architect, or other professional user.[35]

Once plan-view data has been converted to a digital vector format, it must be processed in order to create the relevant functional DSMs. Digitized vector data (e g., contour elevation strings) alone are not well-suited for quantitative analysis by computer.[36] The typically nonsymmetrical form of digitized contour line data complicates its topological ordering for analytical purposes.[37] But the major problem in using a digital representation of contour strings is its uneven density of data and the corresponding uncertainty of elevations at points distant from the digitized lines. Unless a large number of contours at small elevation intervals (frequently absent from grading plans) are to be digitized, the digital vector file must be converted to an appropriate form for representation as a DSM.[38] The remainder of this section will discuss the methods used to convert digitized vector data to a functional DSM. These methods will be grouped into three general categories: (1) regular mesh grids, (2) triangulated irregular networks, and (3) composite mesh grids.

Regular Mesh Grids

The regular mesh grid is the most widely used DSM method today [1993],[39] and some method of converting contours to a grid-based DSM seems essential to any geoprocessing system.[40] As with its manual counterpart, the simplest grid-based DSMs consist of two perpendicular sets of parallel, evenly spaced lines forming a square grid pattern.[41] The intersections of the grid lines (nodes) represent sampling points for which elevations are interpolated from the digitized data using a wide range of methods (discussed below).

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[34] Leberl and Olson, 616. Vector-based models offer higher efficiency and accuracy for line data when compared to raster-based models (see Clarke, 102-104).
[35] Petrie [*Terrain*], 102-104.
[36] Douglas [*Experiments*], 30 and [*XYNIMAP*], 7.
[37] Douglas [*XYNIMAP*], 7.
[38] Cheng and Idesawa, 582; Goodchild and Lee, 176; Petrie [*Modelling*], 114.
[39] McCullagh [*Digital*], 128.
[40] Douglas [*XYNIMAP*], 7.
[41] Other regular grid patterns include rectangles, triangles and hexagons (see Petrie [*Modelling*], 113). Some methods use "tartan grids," where the grid line spacing is irregular, but where each grid line is included in one of the two parallel sets (see Sabin, 70).

Mesh grids overcome the major limitation of a digitized contour model by providing a uniform density of sampling points.[42] Other advantages of a grid DSM are its simple data structure, ease of implementation, and sophistication of display,[43] but the method has a number of important disadvantages as well.

The most important disadvantage of grid-based DSMs is their inherent limitations in accuracy. Two factors constrain the accuracy of a grid DSM: the grid size and the accuracy of the node interpolations.[44] Because the distribution of grid nodes is unrelated to the distribution of sampled data, any particular grid size may not produce a consistently accurate DSM from data of varying density.[45] One solution is to use a different grid size over different parts of the model so that smaller grids correspond to areas of higher data density, but such a solution has proven to be computationally cumbersome.[46] A simpler solution would be to select a single grid size for the entire data set so as to accurately represent the densest sections of the data. Information and sampling theory suggests that the number of node interpolations should equal the number of digitized data points and the grid spacing should be no more than one-half the size of the smallest feature being modelled.[47] For most digitized data sets, this will be computationally inefficient because areas of low density data are over sampled and processing time is proportional to the number of interpolations required.[48] Regardless of the grid size used, however, a regular mesh grid cannot accurately model irregular features such as breaklines and vertical or near-vertical surfaces.[49]

Grid node interpolation methods are numerous and vary widely, but they can be classified in terms of (1) being either local or global in approach and (2) using either linear or nonlinear interpolation. With linear interpolation, grid node elevations are interpolated on straight line segments fitted to the sample data; with nonlinear interpolation, grid node elevations are interpolated on vertically curved lines. Local methods limit the range of influence on their (linear or nonlinear) interpolations for a given grid node to a subset of the modelled data which is localized around each grid node. The underlying assumption is that a data point's influence on the interpolation of a grid node increases with its proximity to the node, but decreases to zero at some predefined distance from the node.[50] Global methods are more sophisticated than local

[42] Goodchild and Lee, 176; Petrie [*Modelling*] 114.
[43] Douglas [*XYNIMAP*], 7 and [*Experiments*], 30; McCullagh [*Digital*], 140; Mirante and Weingarten, 11; Kennie and McLaren, 254; Milne [*Survey*], 174.
[44] McCullagh [*Digital*], 129.
[45] Petrie [*Modelling*], 113.
[46] McCullagh [*Digital*], 129.
[47] McCullagh [*Digital*], 129; Tobler, 40. Some suggest using the shortest horizontal distance between any two contiguous contours on both the x and y axes (see Gomes, 22-23).
[48] McCullagh [*Digital*], 129; Shearer, 335.
[49] McCullagh [*Digital*], 139; Milne [*Survey*], 174; Mirante and Weingarten, 11; Shearer, 335.
[50] McCullagh [*Digital*], 130.

methods, attempting to fit all the data points to some underlying function; by definition, global methods are nonlinear. Regardless of the available range of interpolation methods, theory recommends that grid node elevations are best interpolated along the line of slope perpendicular to the contours.[51] But, while it is comparatively easy for an experienced human technician to interpret a contour map and determine the best interpolated elevation for any given grid node, defining an automated grid-interpolation algorithm that consistently produces accurate interpolation results is problematic (see Figure 1).

The following discussion of interpolation methods is presented in order of increasing sophistication and difficulty of implementation.

Local methods are simple and easy to implement. The most basic local approach is to simply round off known (digitized) elevation points to the closest corresponding grid nodes, and then use the assigned nodes to interpolate values for the remaining unassigned nodes.[52] Because this approach essentially relocates known data points to grid node locations, its interpolation is based on an approximation of the known data. A more accurate approach would be to honor the known data by (at least) interpolating directly from it.

A number of interpolation algorithms accomplish this by using ***search lines*** through the grid nodes. Search lines intersect digitized elevation lines, capturing values for the subsequent interpolation of a grid node's elevation. A basic search-line strategy would use the two perpendicular sets of parallel grid lines (x and y) to establish search directions. The simplest method would limit the search to two directions on a single line by using only one of the two sets of parallel grid lines. This approach would work well when a search line paralleled the line of slope, but could lead to gross errors in other circumstances. Considering the frequently asymmetrical form of contours, a single-line search is generally unacceptable.

Accuracy can be improved by searching both (x and y) grid lines (four search directions) through each grid node. One algorithm described in the literature searches both grid lines and applies a decision scheme that compares the total search distance for intercepted data on the two grid lines, selecting the grid line with the shortest search distance as the line of linear interpolation.[53] A similar algorithm is used with InSite Software's commercially available earthwork software, *SiteWork* (see Figure 2).[54] This approach increases the likelihood that a search line will follow a slope line through any given grid node,

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[51] Douglas [*XYNIMAP*], 7-8.
[52] Mirante and Weingarten, 11.
[53] Douglas [*Experiments*], 55.
[54] *InSite SiteWork*, 93. The shortest-search-distance decision rule used here is also described in Cheng and Idesawa, 584 and Gomes, 58-60.

especially in the circumstance of more-or-less parallel contours (as in Figure 2), where divergence from the slope line perpendicular to the contours does not result in node-interpolation errors. Nevertheless, this method can still yield node-interpolation errors where parallel contours define ridges or swales diagonal to the grid lines. In the example of Figure 3, this approach would cause node point **P** to be interpolated along grid line **H**, resulting in an elevation error of more than 25 percent at point **P**. The affected area is greater where the contour loops are more extended (see the ***80*** contour in Figure 3).

This problem is eliminated, or at least reduced, when the search along both grid lines is supplemented by also searching the two diagonals to the grid lines (eight search directions). The U.S. Geological Survey's *Contour To Grid Transformation Program* uses this approach, estimating each grid node elevation as a distance-weighted average of the eight elevations captured by the corresponding search lines.[55] Unfortunately, the benefit of increasing the probability of interpolating on the slope line when using eight search directions is limited by averaging all eight of the captured elevations. A better method is found in another contour-to-grid interpolation program which uses the same eight-direction search strategy, but employs "a sequential steepest slope algorithm" to select the optimum line of interpolation (see Figure 3).[56]

Intuitively, the combination of eight search directions and some type of decision scheme should produce better results, over a wider range of data sets, compared to a combination of eight search directions and a weighted averaging scheme. While a weighted average merely approximates a grid node elevation, an appropriate decision scheme should be able to provide a more accurate value based on the given circumstances of a grid node and the corresponding data set. Because a steepest-slope decision rule honors the theoretical ideal of interpolating on the slope line perpendicular to the contours, it seems a better choice than the shortest-distance rule.

Pointwise methods represent a type of local interpolation that avoids the issue of search lines. Rather than being limited to a finite number of search directions, pointwise methods typically assign elevations to grid nodes by using the inverse-weighted average of a subset of the digitized elevations representing a given grid node's *nearest neighbors*. The nearest neighbors can be identified by using (1) an area search within a predefined radius of each grid node, (2) a number-of-neighbors search, usually the nearest five to ten, or (3) a sectored search which places each grid node at the common corner of four

[55] Acevedo, 3. The use of diagonal search lines is also mentioned by Douglas [*Experiments*], 55 and [*XYNIMAP*], 8-10.
[56] Leberl and Olson, 619-620. A steepest slope decision rule is also suggested in Douglas [*Experiments*], 55 and [*XYNIMAP*], 9.

quadrants, each of which are searched for a fixed number (two to four) of nearest neighbors (see Figure 4).[57]

Pointwise methods are normally associated with randomly scattered data sets and may be better suited for use with survey data points rather than digitized elevation lines. Depending on the distribution of a data set, each of the pointwise search methods presents its own set of problems. Area and number-of-neighbors searches can produce satisfactory results with evenly distributed data, but do not perform well with variations (e.g., voids) in the data. Some of these problems are overcome by using a sectored search, but that produces discontinuities of position which are difficult to compensate for.[58] This suggests that pointwise methods are suitable for use on data that has been digitized in a grid pattern but not for data digitized by following the wandering elevation lines typically found on grading plans. In general, when pointwise methods are used with digitized contours, especially in relatively flat areas, the nearest neighbors will tend to be overrepresented by points of equal elevation with the result that many grid nodes will tend to equal the contour elevation.[59] These errors are made graphic in contour maps generated by systems using pointwise methods (see Figure 5).[60] In fact, there are only two circumstances in which such errors might not be expected with digitized contour data: (1) where the terrain is very rough (i.e., there are many closely spaced contours) and (2) where the digitized point spacing along the contours equals the spacing between the contours.[61] This suggests that the problem might also be reduced by processing the digitized contours with a point-reduction algorithm (see Figure 6).[62] Nevertheless, a weighted average merely approximates grid node elevations and such errors would not be expected when using a good search-line interpolation strategy.

Although it too uses a nearest-neighbors, weighted-average approach,[63] ***kriging*** is unique among the local interpolation methods because it produces the only stochastic DSM. Based on the theory of random variables, kriging uses the relationships between digitized data points to develop a statistical description of a surface and then interpolates grid node elevation values in the form of means and variances.[64] A variance is, of course, a measure of error, and the trick is to pick the neighbor weights that yield "the best possible

[57] Clark, 211-214 and Petrie [*Modelling*], 115-116. For a BASIC program source code listing of a subroutine for generating a regular grid using pointwise [inverse square distance weighted] interpolation with the five nearest neighbors, see Milne [*Computer Graphics*], 89, 94-95; for the same in C, see Clarke, 207-210.
[58] Sabin, 74-75. Other problems are discussed by Gomes, 28-30.
[59] Shearer, 331.
[60] The same angularity of contours seen in Figure 5 plagues *Paydirt Sitework*, a commercially available earthwork estimating system marketed by Spectra-Physics [see *Sitework*].
[61] Clark, 216.
[62] The Douglas and Peucker paper describes a number of point-reduction algorithms.
[63] Duckett, 2-10 - 2-13, 3-6.
[64] Sabin, 76.

unbiased estimator" and minimize the variance at each node; kriging attempts this by employing matrix algebra and some fancy inversions and back substitutions.[65] The theoretical ability to minimize the variance is the basis of the claim by kriging's proponents that the method represents "an optimal process of interpolation."[66]
All claims aside, kriging does pose some problems. Depending on the accuracy of the assumptions and estimates involved in the calculations, the variance estimate may be of little value.[67] For example, kriging may produce overly pessimistic variances for grid nodes located within data voids.[68] Kriging assumes that every set of data points is equally representative of the surface and has equal statistical variances[69]; this may be inappropriate for natural data, especially if the goal is to obtain a local model of high precision rather than some type of global average.[70] Several empirical studies seem to substantiate this by demonstrating kriging's inferiority to a number of other interpolation methods.[71] Finally, kriging can be "computationally intractable" unless the volume of data being processed is somehow limited, although such limitation renders the resulting kriged estimates "less optimal."[72] One customary way of limiting the computational load is to krig with a wide grid spacing, followed by some other interpolative method at a higher resolution, but this tends to produce cones at the original data points.[73] These problems would seem to render kriging inappropriate in modeling earthwork for site construction projects.

These local methods can employ either linear or nonlinear interpolation for determining grid node elevations. The primary advantage in using linear interpolation is its ease of implementation: it uses uncomplicated mathematics and is, therefore, computationally inexpensive. On the other hand, linear interpolation is conservative in that it cannot identify surface features clearly inferred by certain vector data sets.[74] Examples of such features include hill tops, sumps, and convex/concave slopes, all of which linear interpolation tends to flatten out. Nonlinear interpolation typically fits a polynomial function to the data set, interpolating the grid nodes on a curve and, therefore, more accurately representing the surface features previously mentioned. This suggests that linear interpolation is better suited to design topography[75] DSMs, where planar surfaces predominate, and nonlinear interpolation is better suited to existing topography DSMs, where non-planar surfaces predominate. The downside of nonlinear interpolation is that it

[65] Duckett, 2-10 - 2-13, 3-6.
[66] Lancaster and Šalkauskas, 271.
[67] McCullagh [*Digital*], 130.
[68] Sabin, 76.
[69] Duckett, 3-2.
[70] Philip and Watson, 753.
[71] Philip and Watson, 753.
[72] Duckett, 3-6 - 3-7.
[73] Philip and Watson, 753.
[74] Philip and Watson, 755.
[75] Exceptions would be parks, golf courses, and other landscaped areas.

greatly increases the computational effort, especially as the degree of the polynomial increases,[76] but this problem is more acute in global interpolation.

Global methods are nonlinear by definition and very complicated. They fit a single surface to all the given data points. The surface is defined by a high-order polynomial whose coefficients are determined by setting up and solving a series of simultaneous equations (one for every given data point). Once the coefficients have been calculated, the resulting polynomials are used to interpolate the grid-node elevations.[77] The problem here is that large data sets increase the number of interpolating points, requiring higher-order interpolating polynomials. This increases both the required computational effort and the risk of "severe oscillations" in the interpolated elevations.[78] One way of addressing this problem is a ***patchwise*** approach that breaks the site into a series of surface patches of equal size and shape, allowing the development of separate lower-order interpolating polynomials for each patch.[79] Nonetheless, both global and patchwise techniques produce less satisfactory results than the simpler (and less-than-perfect) pointwise methods discussed earlier.[80] Finally, global methods do not easily accommodate surface discontinuities (i.e., vertical or near-vertical breaks).[81] Therefore, global methods seem also to be less appropriate for use in site earthwork modeling.

Triangulated Irregular Networks

The increasingly popular triangulated irregular network (TIN) ameliorates both the uneven density problem of digitized elevation strings and the inherent inaccuracy associated with the regular mesh grid. The TIN is simply an irregular triangular mesh whose nodes consist of the actual (digitized) data points. Compared to rectangular mesh grids, its proponents claim that TINs produce "more accurate surface representations with less data storage."[82]

Although early triangulation methods were both unstable (would not produce the same TIN pattern over a series of iterations with a single data set) and extremely slow, current methods are stable, fast, and efficient.[83] Of the current methods, ***Delaunay triangulation*** is considered superior and has gained wide acceptance. Rather than producing an arbitrary TIN, the Delaunay method seeks to form a unique set of equilateral triangles,

[76] Lancaster and Šalkauskas, 129-130.
[77] Petrie [*Modelling*], 116, 117-118.
[78] Lancaster and Šalkauskas, 67, 130; McCullagh [*Digital*], 129-130.
[79] Petrie [*Modelling*], 116-117.
[80] Kennie [*Software*], 157; Shearer, 331, 335.
[81] McCullagh [*Terrain*], 753.
[82] Mirante and Weingarten, 11.
[83] McCuIlagh [*Terrain*], 763 and Petrie [*Modelling*], 119.

with minimum side lengths, and with edges linking natural-neighbor nodes.[84] The intent is to recognize regions of influence surrounding each node, based on the concept of *Thiessen polygons* (also known as Dirichlet, Deltri, Voronoi, or Wigner-Seithz polygons).[85] The Thiessen region of a node is the area of the surface closer to that node than to any other node. Two nodes are natural neighbors "if their respective Thiessen regions share an edge" (see Figure 7).[86] The result is a fast and stable TIN.[87]

The relative efficiency and accuracy of TINs, compared to regular mesh grids, is based on the fact that TINs do not require the interpolation and storage of data at regularly spaced grid node locations; instead, TINs use the actual data points as the only nodes.[88] In a comparison of DSMs created using both TINs and regular mesh grids with eleven different data sets, the TINs consistently showed "lower root mean square errors" (as measured at more than one thousand elevation test points) while using approximately one tenth the number of nodes in the corresponding grid-based DSMs.[89] Another study, comparing contour maps generated from grid- and TIN-based systems to a photogrammetrically measured contour reference, found that TINs "produced consistently better results than random to grid methods," especially when patchwise and global grid techniques were employed.[90] The ability of TINs to accurately model surfaces with fewer nodes than corresponding grid models is due to the direct incorporation of breaklines in the TIN models (see Figure 8), which allows TINs to accurately represent even vertical and near-vertical surfaces and overcomes the major weakness of regular mesh grid DSMs.[91] Nonetheless, if the triangulation algorithm does not honor breaklines, erroneous TIN models can result (see Figure 9).[92]

Although the two studies just mentioned conclude the TIN’S superior accuracy when compared to that of regular mesh grid models, one other study found a TIN model to show greater bias (mean difference) than a grid model based (ironically) on kriging.[93] The bias apparently resulted from the TIN model’s use of linear interpolation because a second TIN model of the same data, but using nonlinear interpolation, was far superior to the kriged grid model. The simplest and most direct interpolation method for TINs is a

[84] Petrie [*Modelling*], 119; Philip and Watson, 755.
[85] Goodchild and Lee, 177; Lancaster and Šalkauskas, 217-218; Macedonio and Pareschi, 859; Sabin, 75.
[86] Goodchild and Lee, 177.
[87] For a source code listing of a C program that performs a Delaunay triangulation, see Macedonio and Pareschi, 863-870; for the same in BASIC, see Philip and Watson.
[88] But the manipulation of a TIN does, however, require more sophisticated algorithms than those needed to manipulate a regular grid McCullagh [*Digital*], 140.
[89] Kumler and Goodchild, 179. It should be noted that, while the number of TIN nodes was one tenth the number of corresponding grid nodes, the TIN data files were no smaller than their grid counterparts.
[90] Kennie [*Software*], 157.
[91] Kennie [*Software*], 162; McCullagh [*Digital*], 139; Milne [*Survey*], 174.
[92] Hogan and Ketteman, 229.
[93] Philip and Watson, 753.

linear combination of each triangle's three nodes, but this results in a surface of jagged, angular plane facets (see Figure 9) and a problem similar to that associated with linear interpolation in mesh grid models: "The interpolation specifies linear data trends directly between natural neighbors and so does not identify convexities or concavities across triangles that can be inferred from trends in the surrounding data."[94] As the study indicated, the problem can be avoided by using nonlinear interpolation; however, this will increase the computation time and is considerably slower than nonlinear interpolation with a regular mesh grid.[95]

Another problem which can produce inaccuracies in TINs is the transitional limitation in the density of triangles across a surface. The problem can be acute on surfaces requiring a rapid transition from low-density to high-density triangles, such as with a relatively flat prairie traversed by a meandering stream. The topographical transition between prairie and stream must either be very gradual or be represented by long and thin triangles. As the Schwartz paradox in using limits to calculate surface areas demonstrates, the resulting long and thin triangles "have a large probability of diverging from the true surface."[96]

Finally, TIN-based DSMs lack the sophistication of display provided by their grid-based counterparts. Specifically, TIN perspective views are relatively ambiguous due to their apparently random and non-uniform triangular patterns, whereas mesh grid perspective views provide unambiguous depth ques in their converging parallel lines and shrinking uniform squares (see Figure 10).[97] This display problem has been overcome in a number of TIN based software packages (e.g., *HASP*, *MOSS*, and *PANACEA*) by interpolating a mesh grid directly from the TIN.[98] Unfortunately, this extra step only adds to the total computation time with these packages.

Composite Mesh Grids

Perhaps due to the previously stated advantages of the regular mesh grid (simple data structure, ease of implementation, and sophistication of display), there has been a demand for techniques -- other than simply decreasing the grid spacing -- to correct the regular mesh grid's inability to accurately represent breakline data. The problem is stated visually in Figure 11, where the lack of a swale breakline causes the grid DSM to flatten the swale and misrepresent the actual terrain. The techniques developed in response to

[94] Philip and Watson, 755; Sabin, 76.
[95] McCullagh [*Digital*], 138 and [*Terrain*], 761.
[96] Douglas [*Experiments*], 31-32.
[97] Kennie and McLaren, 254.
[98] McCullagh [*Digital*], 140; Petrie [*Terrain*], 103. For a source code listing of a C program for the interpolation of a mesh grid from a TIN, see Macedonio and Pareschi, 870-874.

this problem fall into two categories: (1) adding inter-node sampling points, and (2) superimposing the actual breakline data on the mesh grid.

The simplest of these techniques is to increase the number of sampling points between nodes on the grid lines. An example of this is the U.S Geological Survey's *Contour To Grid Transformation Program* (discussed earlier). After all grid-node elevations are calculated, this program uses a "grid adjustment step" that "forces the [grid] surface to pass through given control points" which include "intersections of grid lines with contours and with profile data.[99] Another contour-to-grid algorithm does the same thing by recording as sampling points not just the grid nodes, but also every point where "the mesh lines are intersected with contour lines." All the resulting sampling points are then used to calculate "the sectional shape" corresponding to any given mesh line.[100] Referring again to Figure 11, this approach would yield accurate left-to-right profiles only when those profiles correspond exactly to the grid lines; any left-to-right profiles taken between the grid lines would not accurately reflect the swale indicated by the breakline. While this technique is an improvement over the regular mesh grid, and it would be optimum for volume calculations using cross sections corresponding to the grid lines (to be discussed in a following section), it is not the most accurate solution.

To produce the highest-accuracy grid-based DSMs it is "critical that breakline data can be input and recognized," and "supplementing the regularly spaced data with selected height points at key locations and [with] data strings for breaklines, would appear to be more appropriate solutions."[101] Software packages that have incorporated terrain breaklines have "given further credibility or life to grid-based terrain modelling methods."[102] Examples of such programs include *SCOP DTM* and *HIFI-88*. As the perspective view in Figure 12 demonstrates, *SCOP DTM* superimposes breakline data directly onto the mesh grid; note (in the lower-left corner of the figure) how the breaklines affect the sectional shape of the grid lines between grid nodes. *HIFI-88* provides one of the most thorough incorporations of breakline data: the program starts with a grid structure on which elevation strings (including contour lines, breaklines, and structure lines) are superimposed. The program then integrates the elevation lines and the mesh grid using a triangulation within each grid cell (see Figure l3).[103] By directly incorporating contour lines, breaklines, and other elevation strings with the mesh grid, these programs provide a continuous surface model that directly honors the original data intersected by profiles taken in any direction and at any interval. Looking once more at Figure 11, it is obvious

[99] Acevedo, 3.
[100] Cheng and Idesawa, 583.
[101] Shearer, 335-336.
[102] Petrie [*Modelling*], 118-119.
[103] Petrie [*Impact*], 69.

that inclusion of the swale line with the grid lines would produce an accurate left-to-right profile at any location through the data.[104]

[104] Douglas [*Experiments*], 55-56 underscores the importance of breaklines by describing an extremely efficient "richline model" consisting only of "information rich lines" such as "ridges, channels, and lines of break in slope." Even though the originally digitized contour data is excluded from it, a richline model can still be used to generate an accurate mesh grid model of the original surface.

Surface-to-Surface Volumes

Once DSMs have been created for any two surfaces (e g., original ground and proposed design) using the methods described in the previous section, surface to surface volumes can be calculated. The methods for calculating DSM volumes typically fall into three categories: (1) grid cells, (2) cross sections, and (3) triangular isopaches.

Grid Cells

As it is with the manual methods described earlier, the grid cell method is the simplest way to calculate volumes from DSMs. The grid cell method applied to DSMs is based on a regular mesh grid, with the four nodes of each grid cell serving as the only sampling points. Volumes are computed in the same way as with the manual grid cell method. Although automating the grid cell method with the use of a computer greatly reduces the time required for the calculations, this approach retains the inherent accuracy limitations of its manual counterpart. Examples of software packages using the grid cell method include C.E.I.A.'s *Topographics-II*,[105] CivilSoft's *Site Design Program*,[106] Roctek's *EXPOSÉ Earthwork*,[107] and Spectra-Physics' *Paydirt Sitework*.[108]

Cross Sections

Cross section methods are applicable to all DSMs, whether they are based on regular mesh grids, composite mesh grids, or TINs. These methods use a strategy of either fixed-interval or variable-interval cross sections. With a fixed-interval strategy, the spacing between adjacent cross sections is constant through the entire data set; whereas a variable-interval strategy allows the cross-section spacing to vary through the data set. In any case, coordinate geometry, applied to perimeter points, is "the accepted method" for calculating the end areas of the resulting cross sections.[109]

Fixed-interval methods typically use regular mesh grid DSMs, where the cross-section stations are assigned to one set (x or y) of the DSM's parallel grid lines. The existing and proposed elevations are sampled only at the grid nodes along each cross section (grid) line, and volumes are calculated using the average-end-area method. For any given set of regular mesh grid DSMs, this method will yield approximately the same volumes as the grid cell method and will, therefore, reflect the same inherent inaccuracies of the grid-

[105] This program also offers an option for average-end-area calculations [*C.E.I.A.*].
[106] *Site Design Program*, 4-3.
[107] *EXPOSÉ Earthwork*.
[108] *Paydirt Sitework User's Manual*, viii, 3-11.
[109] Milne [*Computer Graphics*], 18. See pages 26-27 and 53-56 of the same reference for the source code listings of BASIC program subroutines for calculating end areas from coordinates.

node sampling set. Examples of software packages using this cross-section method include C.E.I.A.'s *Topographics-II*,[110] InSite Software's *SiteWork*,[111] and Pinnacle Technology's *Com-Quest*.[112]

In situations where the depths of cut and fill vary widely between corresponding grid nodes at adjacent cross-section lines, the accuracy of the just-described method could be improved by substituting the prismoidal formula for the average-end-area formula. This could be accomplished by applying the formula over every three consecutive cross sections.[113] Accuracy could also be improved by substituting a composite mesh grid for the regular mesh grid in order to increase the number of inter-node sampling points at each cross-section line. Fixed-interval cross sections applied to TIN-based DSMs should provide accuracy comparable to that of fixed-interval cross sections through a composite grid because each cross-section line would intersect all grade breaks along its path. Unfortunately, these improvements seem to be lacking in the currently available fixed-interval cross-section software packages.

"The advantage of *continuous* surface models is that they allow the selection of [cross] section direction and intervals depending on the complexity and irregularity of the surfaces."[114] Therefore, **variable-interval methods** are ideally suited to DSMs based on either TINs or composite mesh grids when they directly integrate and honor digitized elevation line data. In these models, cross sections taken at any spacing and location will intersect not only the grid line data, but also any digitized data along their corresponding alignments.

Some programs, such as Eagle Point Software's *SiteCalc*, allow the user to supplement fixed-interval cross sections by manually selecting locations for additional cross sections at variable intervals.[115] This does not protect against human error, and an automatic method of selecting the optimum location of cross sections would be an improvement.

If volumes were being calculated from TIN-based DSMs, the only logical selection criterion for automatically locating variable interval cross-sections would be the nodes from one or both of the compared TIN surfaces. The optimum interval, and highest accuracy, would be achieved if nodes from both DSMs were used because they would represent the digitized data points (and characterize the grade breaks) on *both* surfaces. Any average-end-area calculation error could then be reduced by supplementing the cross

[110] *C.E.I.A.*
[111] *InSite SiteWork*, 93-94.
[112] Land.
[113] Milne [*Computer Graphics*], 41.
[114] Craine, 223 [emphasis added].
[115] *SiteCalc*, 7-7.

sections taken at each node with additional cross sections taken on some appropriate fixed interval, or by using the prismoidal formula if using only node-generated cross sections. Interestingly, none of the surveyed TIN-based systems did this (they used potentially more accurate triangular isopaches, rather than cross sections, for their volume calculations).

Triangular Isopaches

The triangular isopach method is used exclusively with TIN-based DSMs such as those found in the *MOSS* and *HASP* systems.[116] With the simplest method, TIN volumes are calculated by comparing the existing and proposed TIN surfaces to an arbitrary planar reference.[117] A project's net cut or fill can be determined by summing and netting the volumes of the resulting individual triangular prisms corresponding to the existing and proposed TIN surfaces. But a more sophisticated approach is used in the *HASP* system which generates an "intersurface" TIN through direct comparison of the existing and proposed TIN surfaces. The nodes of the intersurface TIN include those from both the existing and proposed TINs, plus the points of intersection between any triangle boundary on one surface and a breakline on the surface. Each intersurface node represents the elevation differential (depth of cut or fill) between the existing and proposed surfaces at that point. Cut and fill volumes are calculated for the resulting intersurface triangular prisms and totaled for the project.[118]

In tests conducted by the developers of the *HASP* system, the triangular isopach method proved to be inherently more accurate than either grids or cross sections (but, as the line spacing decreased with grids and cross sections, the volumes from all three methods did converge).[119] Other, independent, tests have shown volumes from TIN-based systems to be more accurate than those from grid-based systems.[120] It seems unlikely that these tests made comparisons with systems using variable-interval cross sections and composite mesh grid DSMs. Also, these triangular isopach methods are calculation intensive -- especially when employing nonlinear interpolation -- and do not provide the visual feedback of cross sections during the volume calculations.

[116] Craine, 223; Hogan and Ketteman, 234.
[117] For the source code listing of a C program for calculating TIN volumes against a planar reference, see Macedonio and Pareschi, 861, 873-874.
[118] Hogan and Ketteman, 233-234.
[119] Hogan and Ketteman, 235.
[120] Milne [*Survey*], 176.

Adjustments

Regardless of which digital method is used to calculate cut and fill volumes, the same adjustments described in the section on manual methods must be made. All the software packages surveyed here allow adjustments for strata subtotals. Strata DSMs are generated from borehole data digitized from the grading plan. Eagle Point's *SiteCalc* is unique in that it allows two options for generating strata DSMs. A ***parallel*** option interpolates a strata surface that parallels the original ground surface, whereas an ***absolute*** option interpolates a strata surface independent of the original ground surface.[121] This is a useful feature because many near-surface soils might be expected to parallel the ground surface, while others (e.g., rock) might not.

Of the software packages studied here, all appear to also allow adjustments for horizontal subtotals and shrink/swell factors, but only CivilSoft's *Site Design Program* allows an adjustment for subsidence.[122] Adjustments for topsoil stripping quantities were mostly limited to those packages designed specifically for contractors. InSite Software's *SiteWork* package is unique because it allows stripping depths to be defined in terms of either the uppermost original ground strata or a constant depth from the original ground surface based on a digitized perimeter.[123] Using the uppermost strata would be particularly useful for projects where topsoil depths varied widely over the site. Finally, all the systems would allow vertical subtotals if the user desired to run a series of separate volume calculations against a set of fixed-elevation DSMs.

Graphical Display

One of the major advantages of applying digital methods to earthwork analysis is the ability to easily manipulate the resulting DSM databases to generate various graphical representations of a project. Three-dimensional (or four-dimensional, with color contour shading) mesh grid perspective views are invaluable "for gaining insight into area structures, illustrating spatial relationships, deriving relationships or trends and communicating vast amounts of information at a glance." By employing vertical exaggeration, rotation, and zoom options, perspective views allow the user to inspect the DSM and identify errors from many different viewpoints.[124] All the packages surveyed here generate perspective (or at least isometric) views, but the number of options and quality of display vary widely.

[121] *SiteCalc*, 5-28.
[122] *Site Design Program*, 3-3.
[123] *InSite SiteWork*, 31.
[124] Milne [*Survey*], 176. For a source code listing of a BASIC program for generating 3-D isometric views, including zoom, rotate, and vertical exaggeration, see Milne [*Computer Graphics*], 121-127.

Another graphical view for checking and visualizing a DSM are profiles and cross sections, which are particularly useful for visualizing the vertical relationships between two or more surfaces.[125] Most of the surveyed packages allow profiles and/or cross sections to be displayed on screen.

Graphical Editing Features

Graphical editing features allow the interactive correction of errors identified in the DSM database. Such errors may include the inaccurate location of point data, data points tagged with incorrect elevations, or insufficient data entry. Corrections can be made either on individual data points or on logically connected strings of data points.[126] Major changes can be made in a cut-and-paste mode where a polygon is digitized around the elements to be modified. Once selected, groups of points can be raised, lowered, moved, or deleted. In addition to correcting data-entry errors, these same editing features also allow "contours, slopes or building pads to be raised or lowered to optimize grading operations."[127] All the surveyed systems allow interactive editing in a tabular format and many of them allow this feature in an on-screen, plan-view graphical mode. An improvement could be realized by allowing interactive editing in a three-dimensional graphical mode, a feature apparently lacking in all the packages reviewed here.

[125] Craine, 223.
[126] McCullagh [*Digital*], 141-142.
[127] Hogan and Ketteman, 236.

Conclusions (The Optimum Specification [in 1993])

In general terms, the ideal system should provide high performance at a reasonable price. Performance is defined here in terms of flexibility in data entry and editing, consistently high accuracy, speed and efficiency in processing, and high-quality graphical feedback. These criterions can be met by a system consisting of an electronic digitizing tablet, 486DX-class personal computer with fast VGA (or SVGA) graphics, color printer, and the specified software. The following sections discuss the software specification.

Data Entry

Due to the limitations of raster scanning, primary data input from the grading plan should be by manual digitizing via an electronic digitizing tablet. Digitized data should be in the form of elevation strings (of either constant or variable elevations) and spot (single-point) elevations, combinations of which can describe any conceivable topography represented in plan view. Elevation strings are ideal for proposed design topographies which normally consist of building, paving and other structure outlines, ridge and swale lines, and contour lines. Elevation strings are also ideal for original ground topographies delineated by contour lines. Spot elevations are most appropriate for entering subsurface borehole data, entering original ground topographies delineated by spot survey points, and adding detail (e.g., hilltops and depressions) to elevation string data. Secondary data input should also include the ability to import ASCII, .DXF, and other data files in order to minimize the need for manual digitizing when topographic survey and CAD system data files are available for a particular project.

Interpolation and Modeling

Interpolation and modeling for original ground and proposed design surfaces should be based on a composite mesh grid. Grid nodes should be interpolated with a local search-line strategy, combining eight search directions and an appropriate set of decision rules. For the sake of speed, most interpolations should be linear but there may be some circumstances where nonlinear interpolation would be more appropriate (e.g., hilltops and depressions). Linear and nonlinear interpolation could be user-selected by entering *soft* and *hard* elevation strings. Linear interpolation would be used where search lines intersect hard elevation lines, whereas nonlinear interpolation would result from intersections with soft elevation lines.

By directly incorporating digitized elevation strings, the composite mesh grid DSM would provide a continuous surface model that directly honors the data wherever it occurs. This approach should provide as good (or better) a model as that represented by a

TIN. The best implementation of a TIN for representing original ground and proposed design surfaces requires nonlinear interpolation over the entire surface and still requires the generation of a mesh grid for display purposes. This, combined with the TIN's more complicated data structure and implementation, leaves the composite mesh grid as the best choice for modeling the dense data associated with original ground and proposed design surfaces.

A TIN is, however, a good choice for representing subsurface strata layers. In this case, Delaunay triangulation should be used to connect the relatively widely scattered boreholes on a site plan. Nonlinear interpolation is not requisite and an interpolating surface could be generated using linear methods. At the user's choice, based on the characteristics of the modelled strata, the interpolation should be either dependent or independent of the original ground surface.

The system should also use triangulation with spot elevations entered for the original ground and proposed design surfaces. Triangulation would be used to *tie off* spot elevations to the nearest elevation strings so as to provide proper intersection lines during search-line interpolation for the grid nodes.

Volume Calculations and Adjustments

Volumes should be calculated from parallel, variable-interval cross sections generated at every digitized data point (for all surfaces -- design, existing, and strata) and at each grid line, using the average-end-area formula. Generating a cross section through every data point of all surfaces forces a cross section at every grade break, while generating additional cross sections through the grid lines would serve to reduce the error associated with any widely spaced adjacent cross sections having large differences in their corresponding end areas. In other words, the extra cross sections taken at the grid lines should eliminate the need for using the prismoidal formula. Because the cross sections are cut through a composite mesh grid incorporating the digitized data each surface profile (proposed design, original ground, and any strata layers) would intersect not only the grid lines, but also any digitized data along its alignment. For these reasons, this method should provide volumes nearly as accurate as those associated with triangular isopaches, but with fewer computations and greater speed. Finally, the automation of this method makes it practical for all projects, even parks and golf courses that have been, traditionally, handled more effectively with the manual contour-slice method.

The system should allow adjustments to the cut and fill volumes for all the variables discussed earlier in the section on manual methods. Included should be adjustments for topsoil removal quantities, strata subtotals, shrink/swell factors, subsidence factors, and horizontal/vertical subtotals. Topsoil quantities should be calculated from the cross sections during the volume calculations and would, therefore, be automatically corrected for variations in slope.

Graphical Display and Editing

In general, current industry trends suggest that the ideal system would operate under a *graphical user interface* (GUI) such as Microsoft® Windows. In terms of specific functions, the system should feature fast, high-resolution color graphics for the display of tabular data, plan-view graphics, three-dimensional perspective views, and profile/cross-sectional views. Zoom and pan features should be available in all graphical views. Vertical exaggeration should be available in perspective and profile/cross-sectional views. And rotation, tilt, line overlay, color shading, and solid modeling features should function with perspective views. During volume calculations, the user should have the option of viewing each cross section on the screen in order to realize one of the major advantages of the cross-section method: its graphical feedback.

Interactive editing features should include all the options discussed earlier. Both globally and individually, the user should be able to edit the elevation and position of any data point. Interactive graphical editing should be available in both a two-dimensional plan-view mode and a three-dimensional perspective mode (under a GUI, both views could easily be displayed at the same time).

If such a system were developed and offered for a reasonable price, it could find a very large market among earthmoving contractors, general contractors, civil engineers, and topographic surveyors.

Appendix of Figures

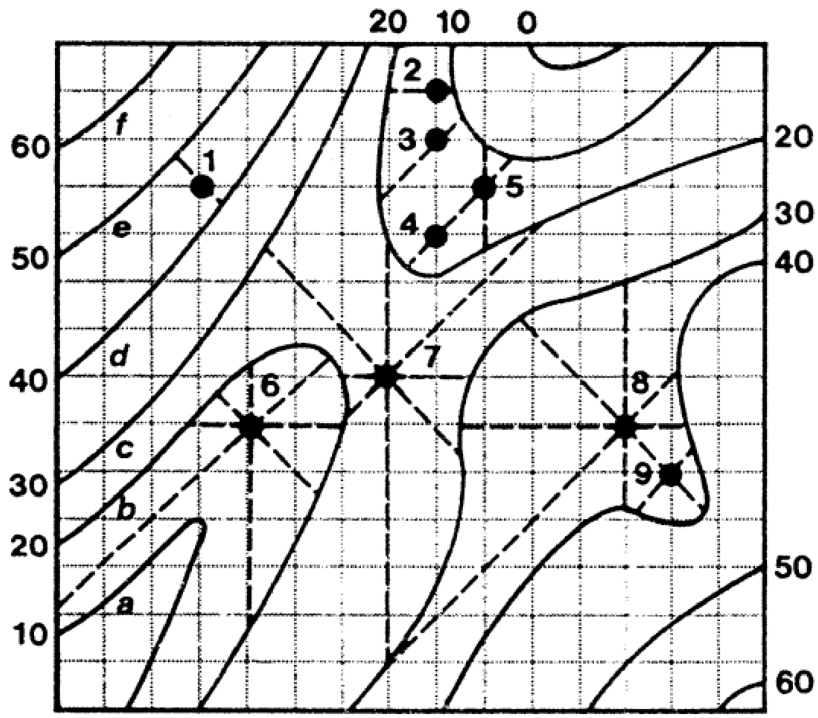


Figure 1 The "Algorithmic Interpolation Problem", where grid nodes 1-5 present a relatively simple interpolation problem compared to that of grid nodes 6-9 [Adding breakline strings to the contours could simplify the interpolation of grid nodes 6-9]. *(Source: Douglas [XYNIMAP], 8.)*

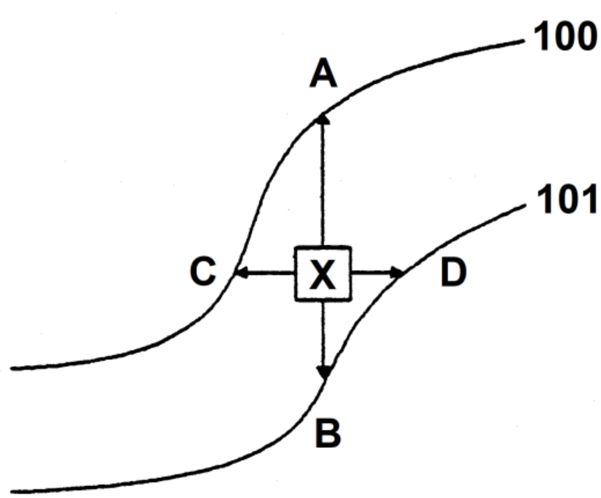


Figure 2 Using four search directions (X-A, X-B, X-C, and X-D) and a shortest-search-distance decision rule, the grid node (X) is interpolated using the C-X-D search alignment. *(Source: InSite SiteWork, 93.)*

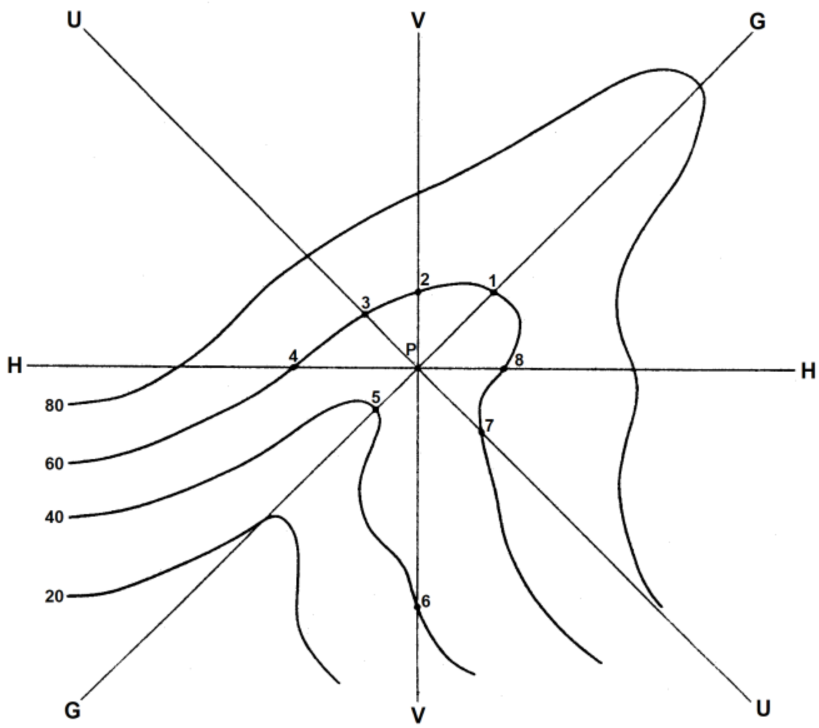


Figure 3 Using eight search directions and a steepest-slope decision rule, the grid node (P) is interpolated using search alignment G-G (Points 1 and 5). *(Source: Leberl and Olson, 620.)*

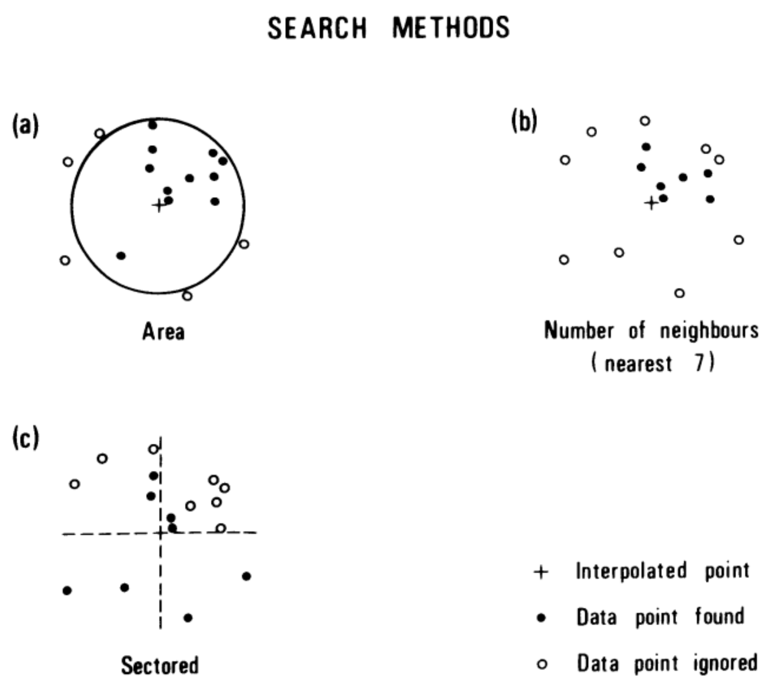


Figure 4 Differing search results when applying various nearest-neighbor search strategies to the same data set. *(Source: Petrie [Modelling], 116.)*

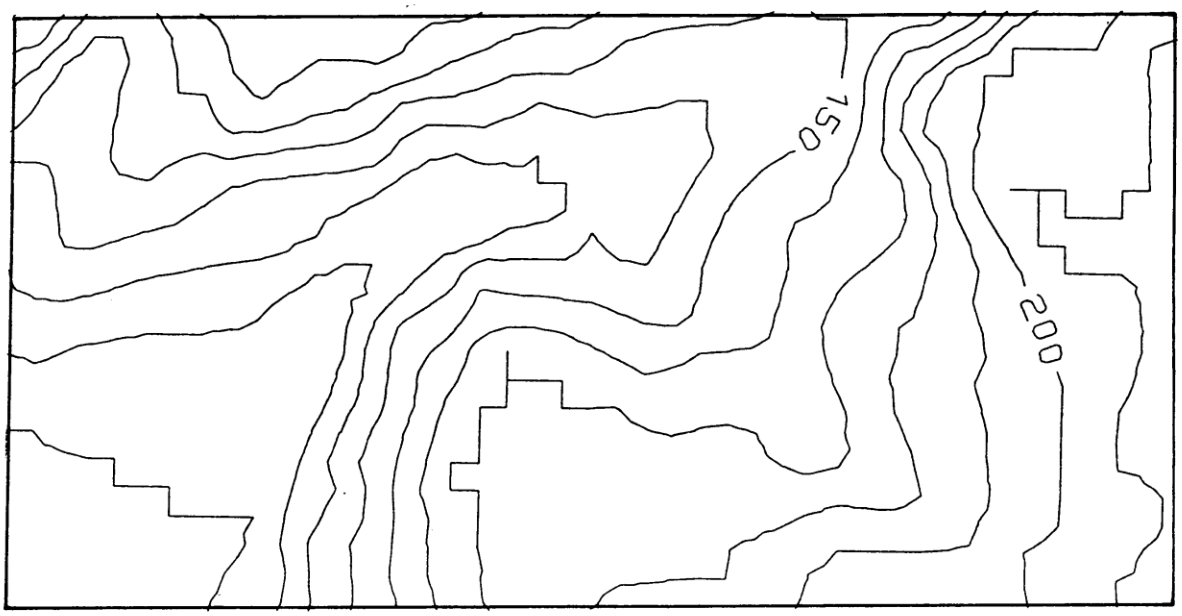


Figure 5 Angular contours resulting from pointwise interpolation. *(Source: Shearer, 332.)*

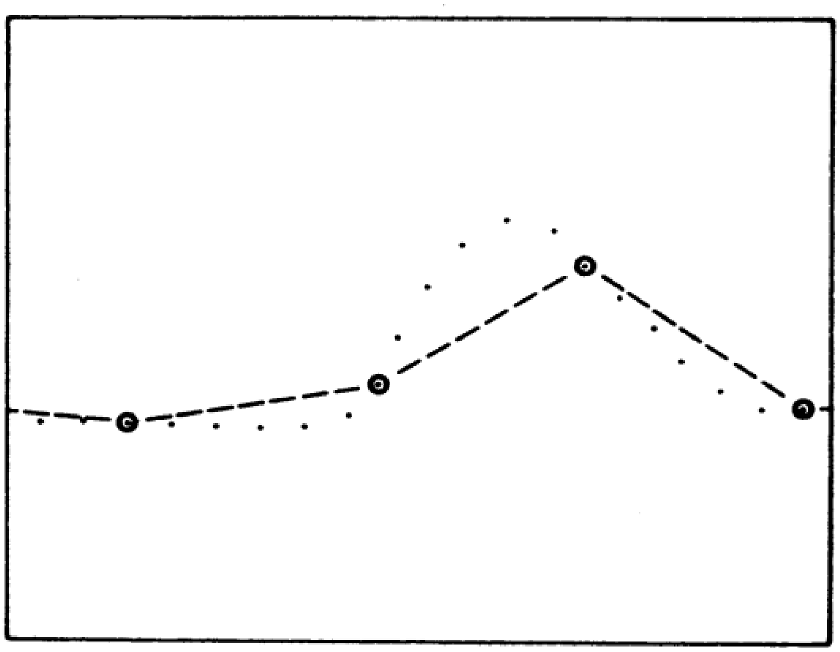


Figure 6 The simplest point-reduction method eliminates all but every $\boldsymbol{n^{th}}$ data point (every 6^{th} point in the example above) but more sophisticated approaches do better at honoring the horizontal fidelity of the original data line. *(Source: Douglas and Peucker, 115.)*

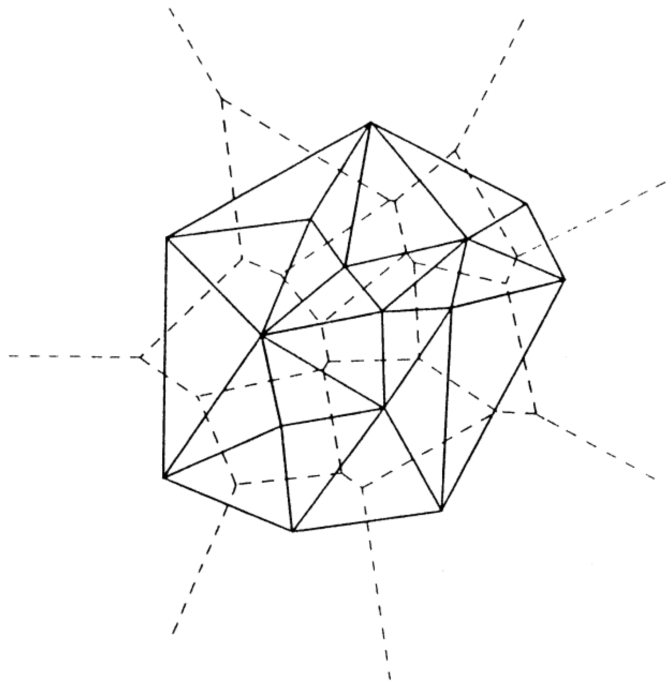


Figure 7 Triangulated irregular network (solid lines) and its implied Thiessen regions (dashed lines) resulting from Delaunay triangulation. *(Source: Macedonio and Pareschi, 860.)*

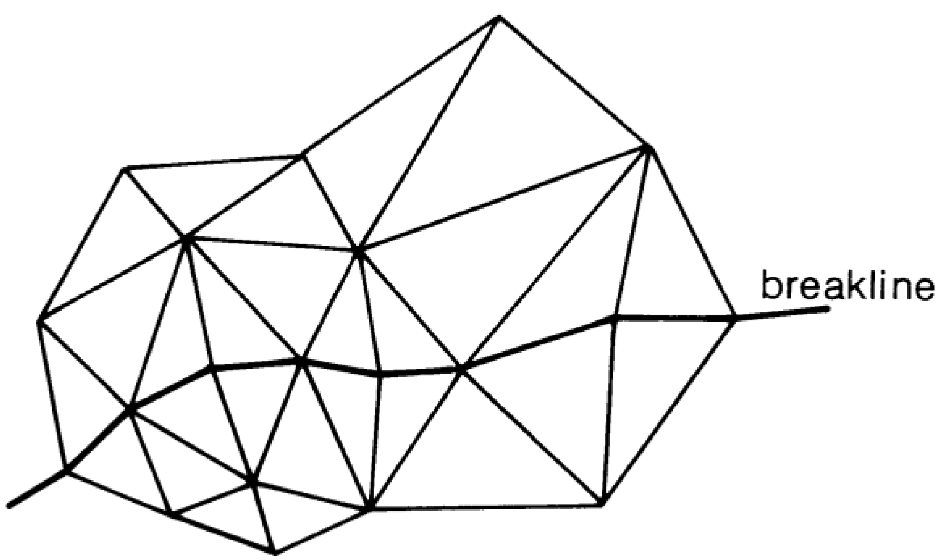


Figure 8 Triangulated irregular network directly incorporating a breakline. *(Source: Kennie [Software], 159.)*

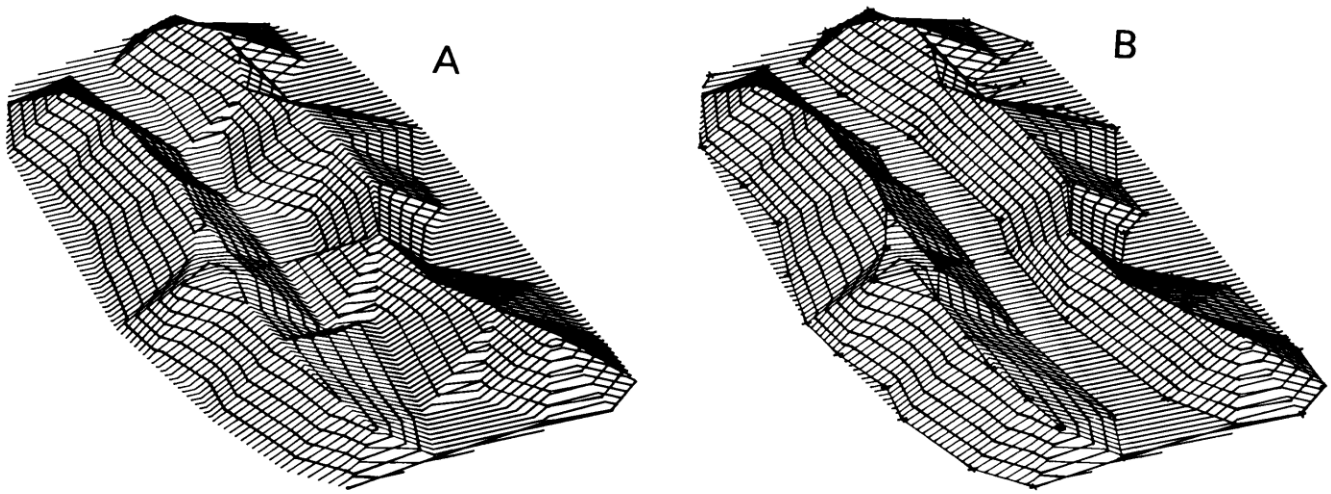
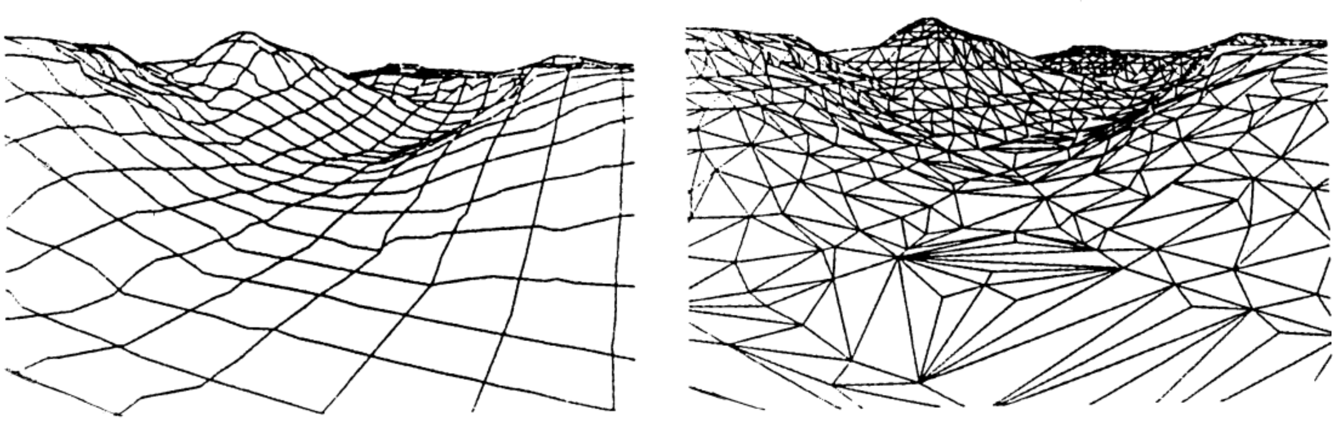


Figure 9 Three-dimensional projections of a TIN-based surface model demonstrating the dramatic difference when the surface excludes (**A**) and includes (**B**) direct incorporation of breaklines. The TIN's angular plane facets are visible in both projections. *(Source: Hogan and Ketteman, 229.)*



(a) (b)

Figure 10 Three-dimensional perspective view of a surface model using regular mesh grid lines (**a**) or TIN lines (**b**) for rendering the view. Grid lines provide the better perspective in this comparison. *(Source: Kennie and McLaren, 254.)*

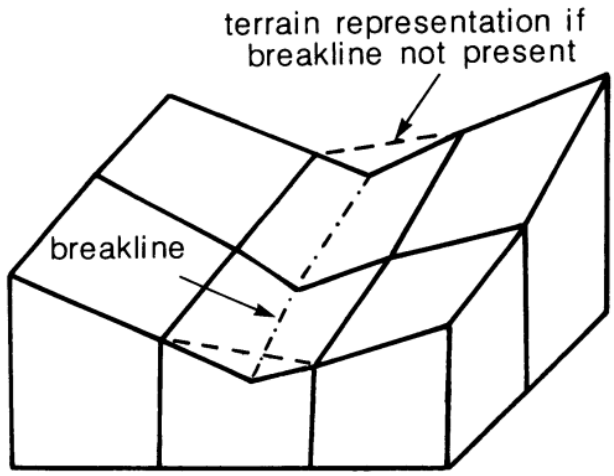


Figure 11 Three-dimensional composite mesh grid view illustrating the error resulting from a failure to incorporate breakline data in a grid surface model. *(Source: Kennie [Software], 158.)*

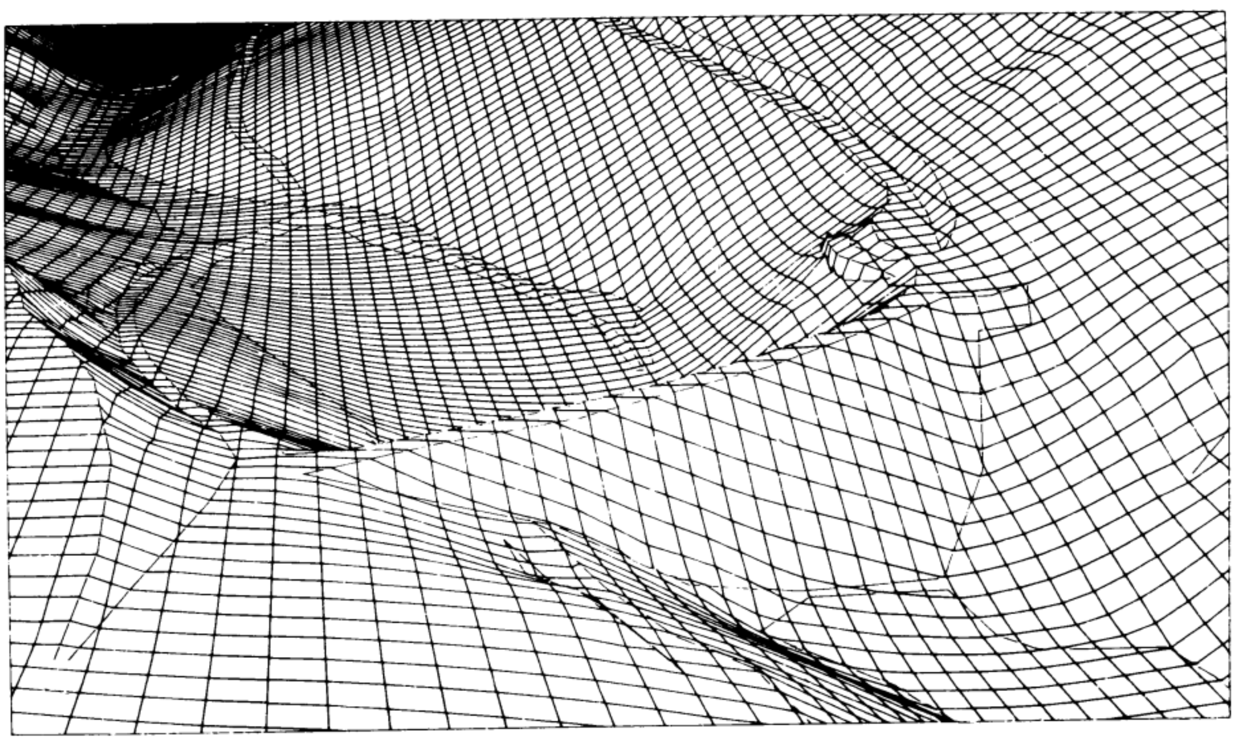


Figure 12 Three-dimensional perspective view reflecting the incorporation of breaklines in a composite mesh grid surface model. *(Source: Petrie [Impact], 68.)*

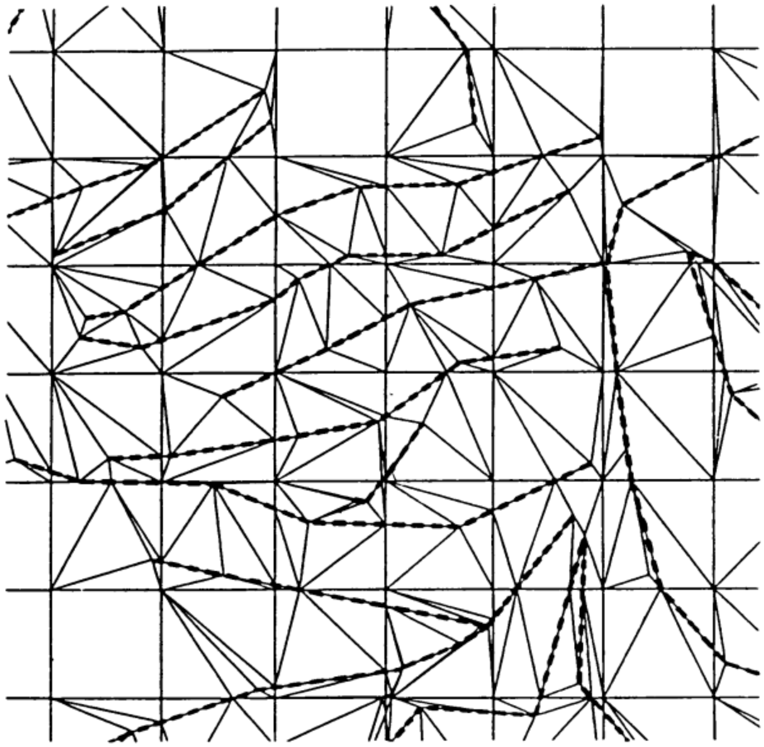


Figure 13 Plan view illustrating the direct incorporation of elevation strings (dashed lines representing contours and/or breaklines) in a composite mesh grid *and* applying triangulation within each grid cell. *(Source: Petrie [Impact], 69.)*

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